Fitting Fish Commodities Data to an Inverse Almost Ideal Demand System

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Problem

In this report, we will fit data on four fish commodities caught and sold each year during 1996 to 2012 using an Inverse Almost Ideal Demand System (IAIDS). This demand system is of the form:

$$w_1 = a_1 + \sum_j \gamma_{1j} \ln q_j + \beta_1 \ln Q_1^*$$

$$w_2 = a_2 + \sum_j \gamma_{2j} \ln q_j + \beta_2 \ln Q_2^*$$

$$w_3 = a_3 + \sum_j \gamma_{3j} \ln q_j + \beta_3 \ln Q_3^*$$

$$w_4 = a_4 + \sum_j \gamma_{4j} \ln q_j + \beta_4 \ln Q_4^*$$

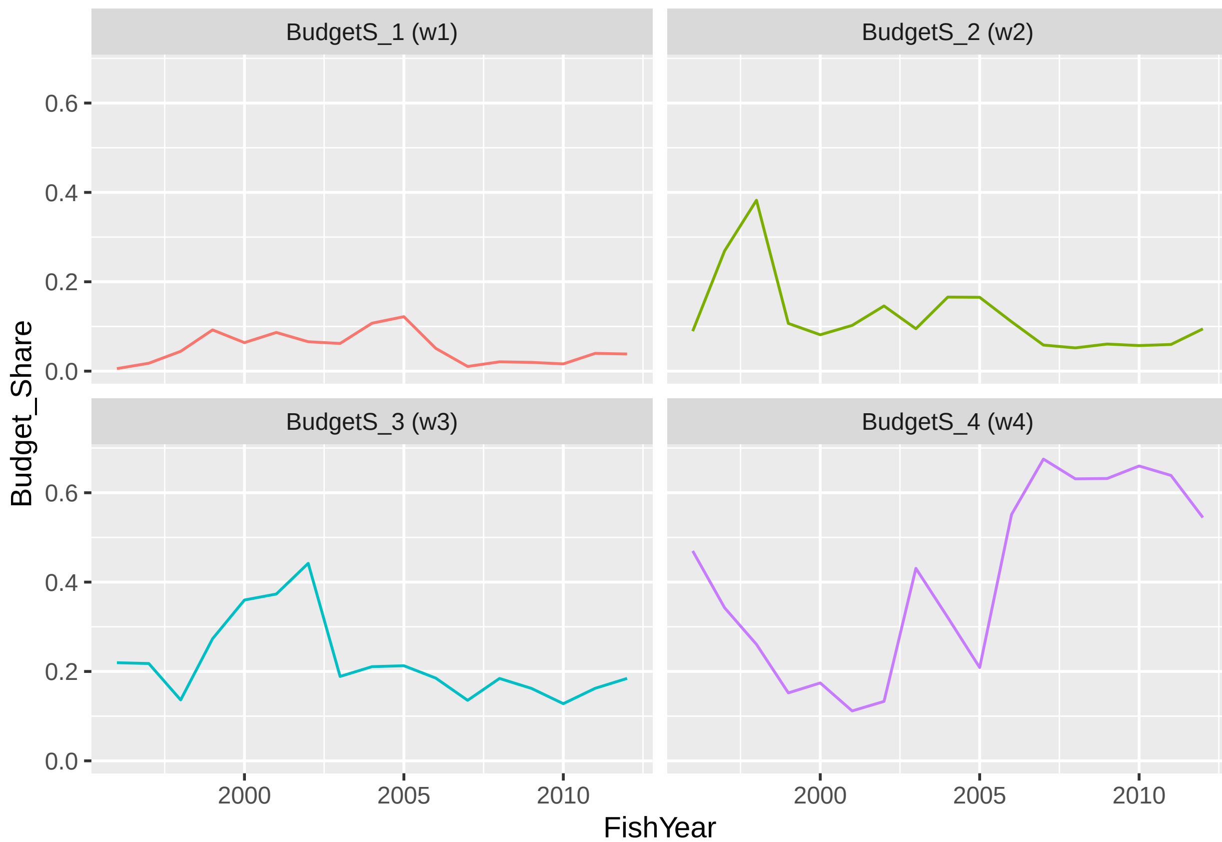
where w_i represents budget shares of commodities for $i = 1, 2, 3, 4$ and q_j represents the quantities of the four commodities each year for $j = 1, 2, 3, 4$. Additionally, $\ln Q_i^*$ represents the stone quantity index of fish commodity i in a year which can be written as $\ln Q_i^* = \sum_j w_j \ln q_j$.

Summary

Due to the stone quantity index of fish commodity being present in each equation, it's plausible the error terms in our model will be contemporaneously correlated. As such, we employed the Seemingly Unrelated Regression (SUR) method for parameter estimation because it effectively accounts for the correlated error terms across equations, offering asymptotically efficient estimates. While initially this method seemed appropriate, we shall find using Weighted Least Squares (WLS) methods for parameter estimation provides a better and more parsimonious model. This is in part due to the low sample size ($n = 17$) and weak off-diagonal residual correlations. We shall fit this system of four equations simultaneously using the `systemfit` package and the `SUR` and `WLS` parameter modifiers for each model. The level of significance used in our analysis is 0.05.

Exploratory Data Analysis

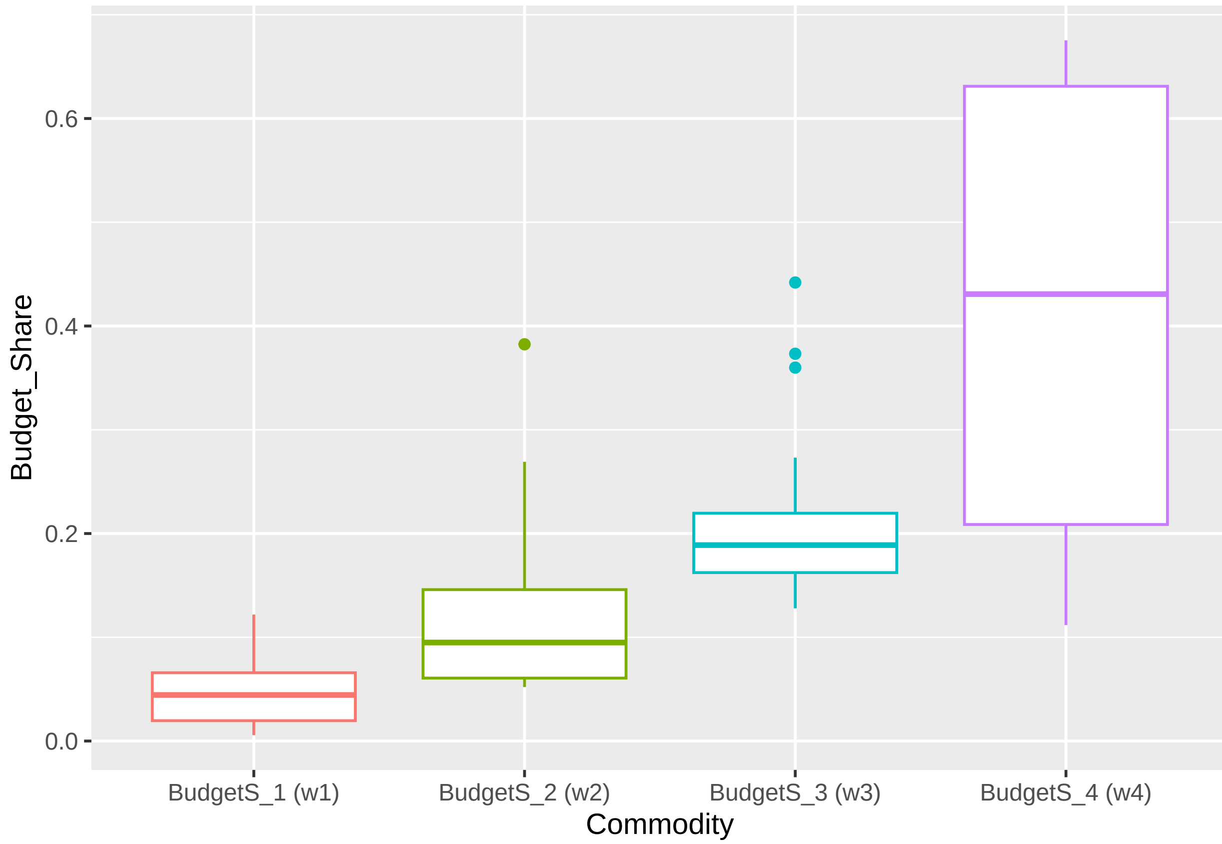
```r
data <- as.data.frame(read.csv("db_test.csv", check.names = FALSE))
long_data <- data %>%
  pivot_longer(
    cols = starts_with("BudgetS_"),
    names_to = "Commodity",
    values_to = "Budget_Share"
    )

ggplot(long_data, aes(x = FishYear, y = Budget_Share, color = Commodity)) +
  geom_line() +
  facet_wrap(~ Commodity) +
  theme(legend.position = "none")
```




```r
any(is.na(data))
```

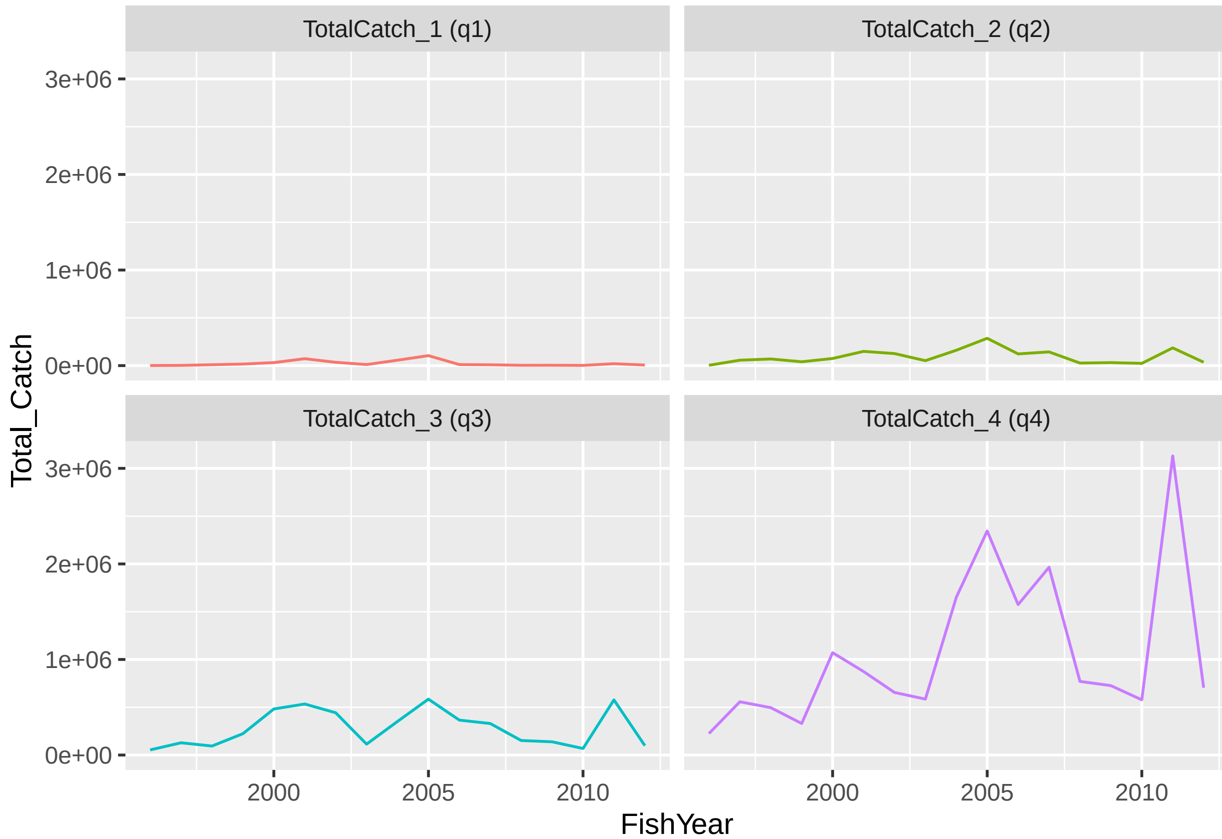
```
## [1] FALSE
```

```r
ggplot(long_data, aes(x = Commodity, y = Budget_Share, color = Commodity)) +
  geom_boxplot() +
  theme(legend.position = "none")
```

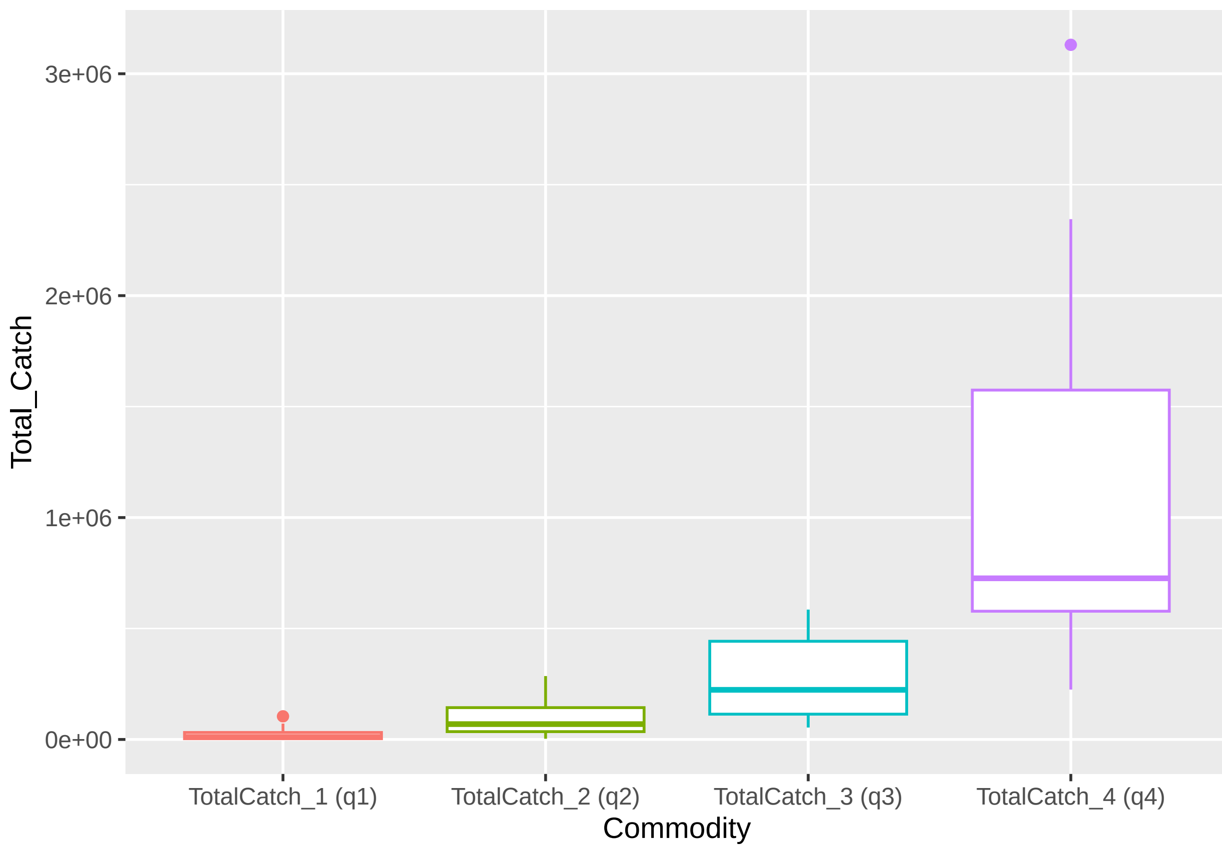


Examining the dataset provided, we see no anomalies in the budget shares over time, no presence of missing values and minimal outliers.

```
long_catch_data <- data %>%
  pivot_longer(
    cols = starts_with("TotalCatch_"),
    names_to = "Commodity",
    values_to = "Total_Catch"
  )

ggplot(long_catch_data, aes(x = FishYear, y = Total_Catch, color = Commodity)) +
  geom_line() +
  facet_wrap(~ Commodity) +
  theme(legend.position = "none")
```




```r
ggplot(long_catch_data, aes(x = Commodity, y = Total_Catch, color = Commodity)) +
  geom_boxplot() +
  theme(legend.position = "none")
```



Similarly, no anomalies detected for the catch data. The varied fish catch data for `Total_Catch_4 (q4)` could be due in part to the varying budget shares for that commodity and specifically we see an increase in budget shares after the year 2005. As such, we can proceed with our analysis in data fitting.

Data fitting

Using the equations outlined in the intro, we must apply ln transformations to our variables defined as `log_q`. Additionally, we shall calculate the stone quantity index of fish commodity defined as `log_Q_star`.

```
data <- data %>%
  mutate(log_q1 = log(`TotalCatch_1 (q1)`),
         log_q2 = log(`TotalCatch_2 (q2)`),
         log_q3 = log(`TotalCatch_3 (q3)`),
         log_q4 = log(`TotalCatch_4 (q4)`))

data <- data %>%
  mutate(log_Q_star = `BudgetS_1 (w1)` * log_q1 + `BudgetS_2 (w2)` * log_q2 +
           `BudgetS_3 (w3)` * log_q3 + `BudgetS_4 (w4)` * log_q4)
```

We chose the SUR method for parameter estimation because the model includes the stone quantity index of the fish commodity, which links the equations together through the relationship $\ln Q_i^* = \sum_j w_j \ln q_j$. This relationship implies that changes in the stone quantity index affect multiple equations simultaneously, as the index is a common explanatory variable across the system. As such, the error terms of the equations are likely to be contemporaneously correlated, as unobserved factors influencing one equation may also impact the others. The SUR method is specifically designed to handle such systems, accounting for these correlations to produce efficient parameter estimates.

We can use the `systemfit` package to simultaneously fit the data to the set of equations in the IAIDS with the `SUR` modifier for the estimation method. Each equation we are fitting is defined as `w1`, `w2`, `w3`, and `w4`.

```r
w1 <- `BudgetS_1 (w1)` ~ log_q1 + log_q2 + log_q3 + log_q4 + log_Q_star
w2 <- `BudgetS_2 (w2)` ~ log_q1 + log_q2 + log_q3 + log_q4 + log_Q_star
w3 <- `BudgetS_3 (w3)` ~ log_q1 + log_q2 + log_q3 + log_q4 + log_Q_star
w4 <- `BudgetS_4 (w4)` ~ log_q1 + log_q2 + log_q3 + log_q4 + log_Q_star

system <- list(eq1 = w1, eq2 = w2, eq3 = w3, eq4 = w4)

fitSUR <- systemfit(system, method = "SUR", data = data)
summary(fitSUR, residCov = FALSE, equations = TRUE)
```

```
##
## systemfit results
## method: SUR
##
##         N DF      SSR detRCov   OLS-R2 McElroy-R2
## system 68 44 0.124978       0 0.869419   0.996972
##
##      N DF      SSR      MSE     RMSE       R2   Adj R2
## eq1 17 11 0.002091 0.000190 0.013787 0.894963 0.847218
## eq2 17 11 0.052952 0.004814 0.069382 0.556887 0.355472
## eq3 17 11 0.021996 0.002000 0.044717 0.829859 0.752522
## eq4 17 11 0.047939 0.004358 0.066016 0.930362 0.898709
##
##
## SUR estimates for 'eq1' (equation 1)
## Model Formula: `BudgetS_1 (w1)` ~ log_q1 + log_q2 + log_q3 + log_q4 + log_Q_star
##
##                 Estimate   Std. Error  t value Pr(>|t|)
## (Intercept) -0.064406138  0.099104382 -0.64988 0.529107
## log_q1       0.026234993  0.008612343  3.04621 0.011123 *
## log_q2      -0.000320224  0.008432297 -0.03798 0.970387
## log_q3      -0.015752701  0.010517636 -1.49774 0.162336
## log_q4       0.011456514  0.015423663  0.74279 0.473175
## log_Q_star  -0.008638002  0.006029513 -1.43262 0.179767
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.013787 on 11 degrees of freedom
## Number of observations: 17 Degrees of Freedom: 11
## SSR: 0.002091 MSE: 0.00019 Root MSE: 0.013787
## Multiple R-Squared: 0.894963 Adjusted R-Squared: 0.847218
##
##
## SUR estimates for 'eq2' (equation 2)
## Model Formula: `BudgetS_2 (w2)` ~ log_q1 + log_q2 + log_q3 + log_q4 + log_Q_star
##
##               Estimate Std. Error  t value  Pr(>|t|)
## (Intercept)  0.8966763  0.4987320  1.79791 0.0996625 .
```

```
## log_q1      -0.0327238  0.0433407 -0.75504 0.4660863
## log_q2       0.1343682  0.0424346  3.16648 0.0089748 **
## log_q3      -0.0897468  0.0529289 -1.69561 0.1180435
## log_q4      -0.0415478  0.0776179 -0.53529 0.6031052
## log_Q_star  -0.0278026  0.0303429 -0.91628 0.3791700
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.069382 on 11 degrees of freedom
## Number of observations: 17 Degrees of Freedom: 11
## SSR: 0.052952 MSE: 0.004814 Root MSE: 0.069382
## Multiple R-Squared: 0.556887 Adjusted R-Squared: 0.355472
##
##
## SUR estimates for 'eq3' (equation 3)
## Model Formula: `BudgetS_3 (w3)` ~ log_q1 + log_q2 + log_q3 + log_q4 + log_Q_star
##
##               Estimate Std. Error  t value  Pr(>|t|)
## (Intercept)  0.5124859  0.3214371  1.59436 0.1391626
## log_q1       0.0282495  0.0279334  1.01132 0.3335938
## log_q2      -0.0221278  0.0273495 -0.80908 0.4356223
## log_q3       0.1506420  0.0341131  4.41596 0.0010354 **
## log_q4      -0.1711732  0.0500254 -3.42173 0.0057058 **
## log_Q_star   0.0175547  0.0195562  0.89765 0.3885958
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.044717 on 11 degrees of freedom
## Number of observations: 17 Degrees of Freedom: 11
## SSR: 0.021996 MSE: 0.002 Root MSE: 0.044717
## Multiple R-Squared: 0.829859 Adjusted R-Squared: 0.752522
##
##
## SUR estimates for 'eq4' (equation 4)
## Model Formula: `BudgetS_4 (w4)` ~ log_q1 + log_q2 + log_q3 + log_q4 + log_Q_star
##
##               Estimate Std. Error  t value Pr(>|t|)
## (Intercept) -0.4707319  0.4745356 -0.99198 0.342525
## log_q1      -0.0216589  0.0412380 -0.52522 0.609855
## log_q2      -0.0990473  0.0403759 -2.45313 0.032067 *
## log_q3      -0.0764598  0.0503610 -1.51824 0.157159
## log_q4       0.1660951  0.0738522  2.24902 0.045969 *
## log_Q_star   0.0824796  0.0288708  2.85685 0.015604 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.066016 on 11 degrees of freedom
## Number of observations: 17 Degrees of Freedom: 11
## SSR: 0.047939 MSE: 0.004358 Root MSE: 0.066016
## Multiple R-Squared: 0.930362 Adjusted R-Squared: 0.898709
```

Analysis

Using the Breusch-Godfrey test we can assess if autocorrelation is present in our error terms. The null hypothesis is that there is no serial correlation of order 1.

```
bg_test_w1 <- bgtest(fitSUR$eq[[1]]$residuals ~ 1, order = 1)
bg_test_w2 <- bgtest(fitSUR$eq[[2]]$residuals ~ 1, order = 1)
bg_test_w3 <- bgtest(fitSUR$eq[[3]]$residuals ~ 1, order = 1)
bg_test_w4 <- bgtest(fitSUR$eq[[4]]$residuals ~ 1, order = 1)
list(bg_test_w1, bg_test_w2, bg_test_w3, bg_test_w4)
```

```
## [[1]]
##
##  Breusch-Godfrey test for serial correlation of order up to 1
##
## data:  fitSUR$eq[[1]]$residuals ~ 1
## LM test = 1.0782, df = 1, p-value = 0.2991
##
##
## [[2]]
##
##  Breusch-Godfrey test for serial correlation of order up to 1
##
## data:  fitSUR$eq[[2]]$residuals ~ 1
## LM test = 0.36095, df = 1, p-value = 0.548
##
##
## [[3]]
##
##  Breusch-Godfrey test for serial correlation of order up to 1
##
## data:  fitSUR$eq[[3]]$residuals ~ 1
## LM test = 0.28956, df = 1, p-value = 0.5905
##
##
## [[4]]
##
##  Breusch-Godfrey test for serial correlation of order up to 1
##
## data:  fitSUR$eq[[4]]$residuals ~ 1
## LM test = 0.77083, df = 1, p-value = 0.38
```

Using our level of significance of 0.05, we can fail to reject the null hypothesis that there is no autocorrelation in the residuals for each individual equation.

Additionally, we shall use the Breusch-Pagan test to determine whether or not heteroscedasticity is present in our error terms. The null hypothesis is homoscedasticity is present.

```
bp_test_w1 <- bptest(fitSUR$eq[[1]]$residuals ~ log_q1 + log_q2 + log_q3 + log_q4 + log_Q_star,
                     data = data)
bp_test_w2 <- bptest(fitSUR$eq[[2]]$residuals ~ log_q1 + log_q2 + log_q3 + log_q4 + log_Q_star,
                     data = data)
bp_test_w3 <- bptest(fitSUR$eq[[3]]$residuals ~ log_q1 + log_q2 + log_q3 + log_q4 + log_Q_star,
```

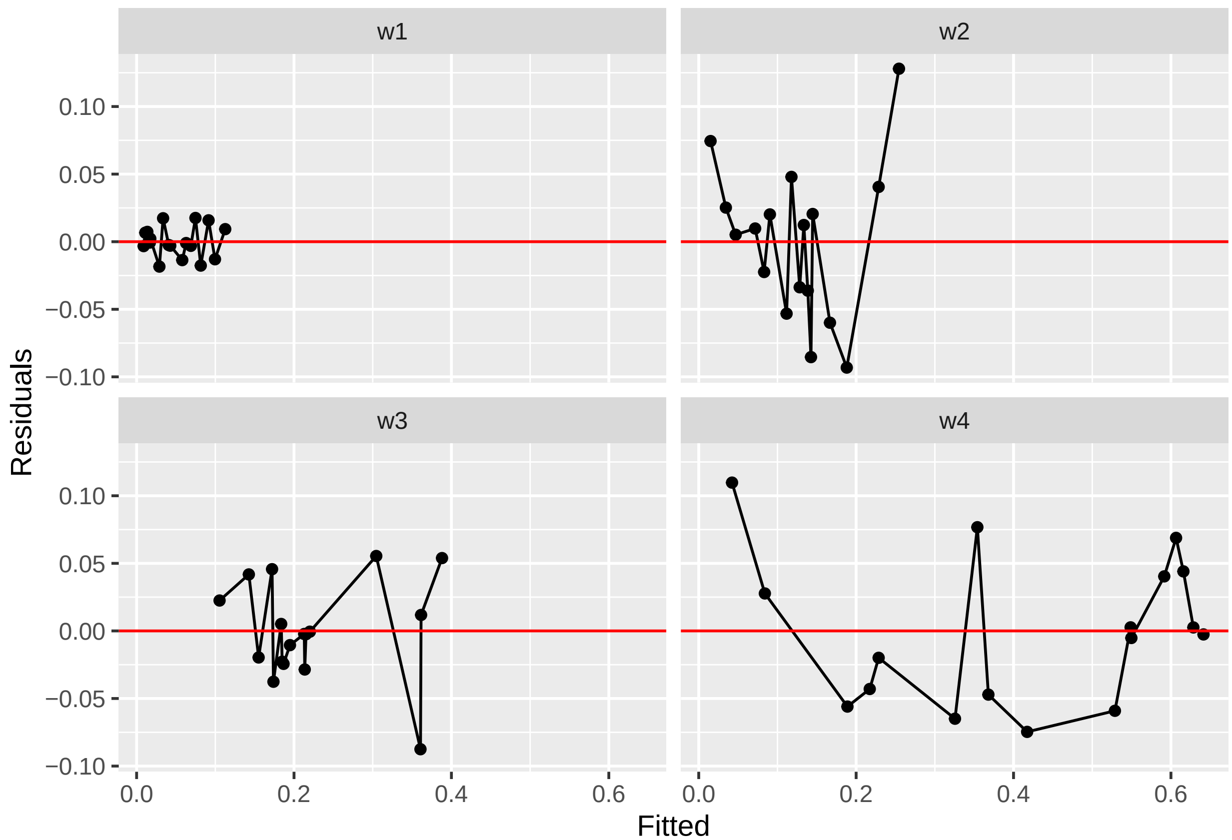
```r
                     data = data)
bp_test_w4 <- bptest(fitSUR$eq[[4]]$residuals ~ log_q1 + log_q2 + log_q3 + log_q4 + log_Q_star,
                     data = data)

list(bp_test_w1, bp_test_w2, bp_test_w3, bp_test_w4)
```

```
## [[1]]
##
##  studentized Breusch-Pagan test
##
## data:  fitSUR$eq[[1]]$residuals ~ log_q1 + log_q2 + log_q3 + log_q4 +     log_Q_star
## BP = 4.8503, df = 5, p-value = 0.4344
##
##
## [[2]]
##
##  studentized Breusch-Pagan test
##
## data:  fitSUR$eq[[2]]$residuals ~ log_q1 + log_q2 + log_q3 + log_q4 +     log_Q_star
## BP = 10.43, df = 5, p-value = 0.06394
##
##
## [[3]]
##
##  studentized Breusch-Pagan test
##
## data:  fitSUR$eq[[3]]$residuals ~ log_q1 + log_q2 + log_q3 + log_q4 +     log_Q_star
## BP = 8.5675, df = 5, p-value = 0.1276
##
##
## [[4]]
##
##  studentized Breusch-Pagan test
##
## data:  fitSUR$eq[[4]]$residuals ~ log_q1 + log_q2 + log_q3 + log_q4 +     log_Q_star
## BP = 9.1801, df = 5, p-value = 0.1021
```

Using our level of significance of 0.05, we can fail to reject the null hypothesis that homoscedasticity is present in our error terms. This can also be seen by plotting the residuals versus fitted.

```r
fitted_w1 <- fitSUR$eq[[1]]$fitted.values
fitted_w2 <- fitSUR$eq[[2]]$fitted.values
fitted_w3 <- fitSUR$eq[[3]]$fitted.values
fitted_w4 <- fitSUR$eq[[4]]$fitted.values

residuals_w1 <- fitSUR$eq[[1]]$residuals
residuals_w2 <- fitSUR$eq[[2]]$residuals
residuals_w3 <- fitSUR$eq[[3]]$residuals
residuals_w4 <- fitSUR$eq[[4]]$residuals

plot_data <- data.frame(
  Fitted = c(fitted_w1, fitted_w2, fitted_w3, fitted_w4),
  Residuals = c(residuals_w1, residuals_w2, residuals_w3, residuals_w4),
```

```
  Equation = rep(paste0("w", 1:4), times = c(17, 17, 17, 17))
)

ggplot(plot_data, aes(x = Fitted, y = Residuals)) +
  geom_point() +
  geom_line() +
  geom_hline(yintercept = 0, color = "red") +
  facet_wrap(~ Equation) +
  theme(legend.position = "none")
```



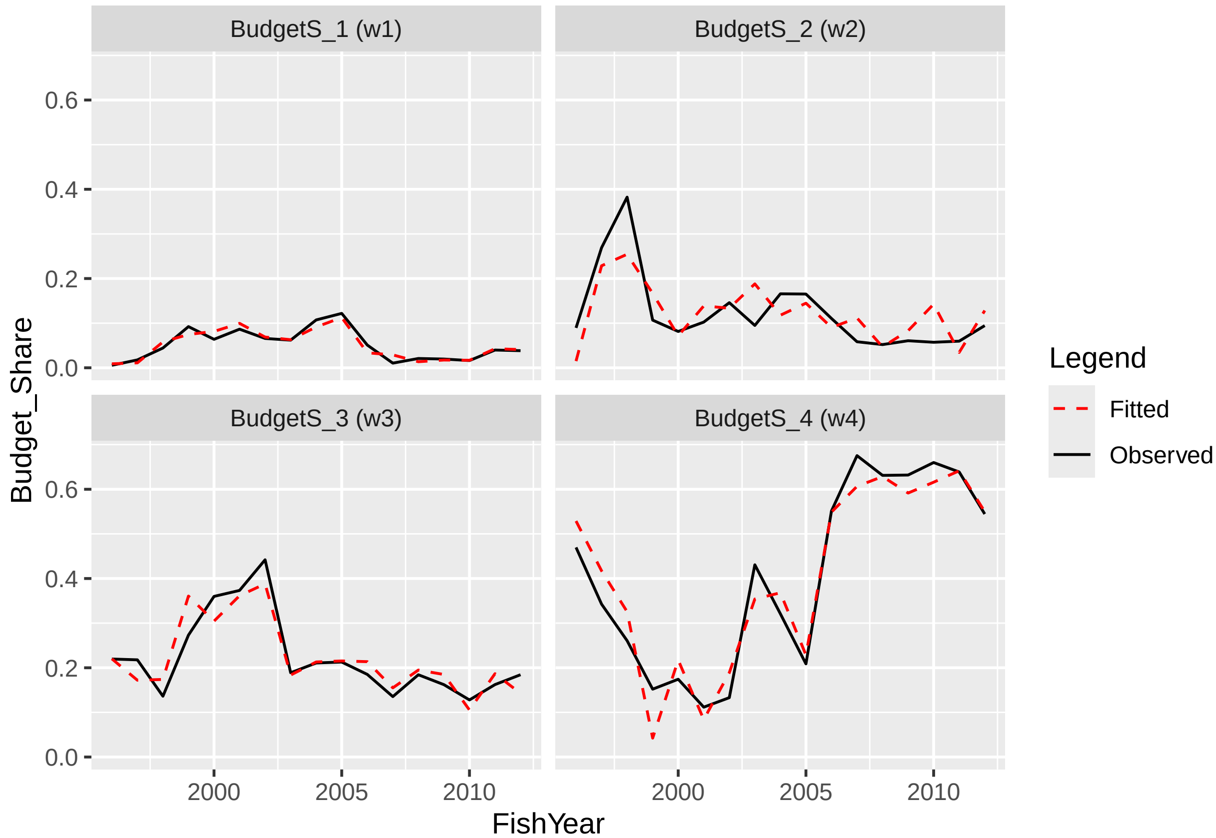
We can see `w2` and `w4` show apparent patterns, suggesting potential violations of independence. Specifically the residuals in `w2` are ballooning outwards which indicates a potential non-constant variance. This is not surprising as the p-value for the Breusch-Pagan test for `w2` is somewhat low at 0.06394. Additionally, we see the residual plots suggests different variances among each equation due to the varied trends. However, all equations are somewhat symmetric around 0. Based on this analysis, we conclude that the disturbances of the individual equations can be assumed to be independent and identically distributed, with the caveat that further investigation into heteroscedasticity in `w2` might be warranted.

```
round(cov2cor(fitSUR$residCov), 2)
```

```
##       eq1   eq2   eq3   eq4
## eq1  1.00 -0.01 -0.35  0.09
## eq2 -0.01  1.00 -0.02 -0.84
## eq3 -0.35 -0.02  1.00 -0.51
## eq4  0.09 -0.84 -0.51  1.00
```

Examining the residual correlation matrix, we can see the `w2` and `w4` have a strongly negative correlation at -0.84. Whereas `w1` and `w3` have a weak negative correlation at -0.35 as well as `w3` and `w4` with a weak negative correlation of -0.51. Lastly, all other equations have little to no correlation existing between each other. The presence of contemporaneously correlated error terms, particularly the strong negative correlation between `w2` and `w4`, further justifies our use of the SUR method for estimation.

```
fitted_data <- data.frame(
  Fitted = c(fitted_w1, fitted_w2, fitted_w3, fitted_w4),
  Commodity = rep(c("BudgetS_1", "BudgetS_2", "BudgetS_3", "BudgetS_4"), times = c(17, 17, 17, 17))
)

long_data <- data %>%
  pivot_longer(
    cols = starts_with("BudgetS_"),
    names_to = "Commodity",
    values_to = "Budget_Share"
  )

long_data <- long_data[order(long_data$Commodity),]

long_data <- long_data %>%
  mutate(Fitted = fitted_data$Fitted)

ggplot(long_data, aes(x = FishYear)) +
  geom_line(aes(y = Budget_Share, color = "Observed")) +
  geom_line(aes(y = Fitted, color = "Fitted"), linetype = 'dashed') +
  facet_wrap(~ Commodity) +
  scale_color_manual(values = c("Observed" = "black", "Fitted" = "red")) +
  labs(color = "Legend")
```



As we can see the fitted model visually aligns with the observed data quite well.

```r
summary(fitSUR, residCov = FALSE, equations = FALSE)
```

```
##
## systemfit results
## method: SUR
##
##         N DF      SSR detRCov   OLS-R2 McElroy-R2
## system 68 44 0.124978       0 0.869419   0.996972
##
##      N DF      SSR      MSE     RMSE       R2   Adj R2
## eq1 17 11 0.002091 0.000190 0.013787 0.894963 0.847218
## eq2 17 11 0.052952 0.004814 0.069382 0.556887 0.355472
## eq3 17 11 0.021996 0.002000 0.044717 0.829859 0.752522
## eq4 17 11 0.047939 0.004358 0.066016 0.930362 0.898709
##
##
## Coefficients:
##                      Estimate   Std. Error  t value  Pr(>|t|)
## eq1_(Intercept) -0.064406138  0.099104382 -0.64988 0.5291066
## eq1_log_q1       0.026234993  0.008612343  3.04621 0.0111226 *
## eq1_log_q2      -0.000320224  0.008432297 -0.03798 0.9703873
## eq1_log_q3      -0.015752701  0.010517636 -1.49774 0.1623356
## eq1_log_q4       0.011456514  0.015423663  0.74279 0.4731748
## eq1_log_Q_star  -0.008638002  0.006029513 -1.43262 0.1797674
```

```
## eq2_(Intercept)  0.896676262  0.498732013  1.79791 0.0996625 .
## eq2_log_q1      -0.032723772  0.043340678 -0.75504 0.4660863
## eq2_log_q2       0.134368224  0.042434618  3.16648 0.0089748 **
## eq2_log_q3      -0.089746788  0.052928856 -1.69561 0.1180435
## eq2_log_q4      -0.041547807  0.077617904 -0.53529 0.6031052
## eq2_log_Q_star  -0.027802561  0.030342866 -0.91628 0.3791700
## eq3_(Intercept)  0.512485869  0.321437057  1.59436 0.1391626
## eq3_log_q1       0.028249529  0.027933438  1.01132 0.3335938
## eq3_log_q2      -0.022127811  0.027349475 -0.80908 0.4356223
## eq3_log_q3       0.150641992  0.034113101  4.41596 0.0010354 **
## eq3_log_q4      -0.171173182  0.050025405 -3.42173 0.0057058 **
## eq3_log_Q_star   0.017554704  0.019556237  0.89765 0.3885958
## eq4_(Intercept) -0.470731904  0.474535647 -0.99198 0.3425249
## eq4_log_q1      -0.021658920  0.041237972 -0.52522 0.6098548
## eq4_log_q2      -0.099047278  0.040375870 -2.45313 0.0320672 *
## eq4_log_q3      -0.076459794  0.050360972 -1.51824 0.1571590
## eq4_log_q4       0.166095062  0.073852212  2.24902 0.0459689 *
## eq4_log_Q_star   0.082479563  0.028870759  2.85685 0.0156042 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The R^2 for all equations are quite high with majority above 0.8, with exception `w2`. In particular `w1` has an R^2 of 0.894963, indicating this equation is the best fit and a RMSE of 0.013787 suggesting high prediction accuracy. On the other hand `w2` has the lowest R^2 of 0.556887 and highest RMSE of 0.069382, indicating the weakest fit among the four equations and less accurate predictions. This is not surprising given the issues of independence and heteroskedasticity seen in the previous analyses. Lastly, McElroy's R^2 for the entire system is 0.996972, indicating a good overall fit as it is very close to 1. This high value suggests that the system collectively explains nearly all of the variance in the observed data.

Conclusion

Using the SUR method of estimation, we simultaneously fit the equations of IAIDS as follows:

$$w_1 = -0.06 + 0.03 \ln q_1 - 0.0003 \ln q_2 - 0.02 \ln q_3 + 0.01 \ln q_4 - 0.01 \ln Q_1^*$$

$$w_2 = 0.90 - 0.03 \ln q_1 + 0.13 \ln q_2 - 0.09 \ln q_3 - 0.04 \ln q_4 - 0.03 \ln Q_2^*$$

$$w_3 = 0.51 + 0.03 \ln q_1 - 0.02 \ln q_2 + 0.15 \ln q_3 - 0.17 \ln q_4 + 0.02 \ln Q_3^*$$

$$w_4 = -0.47 - 0.02 \ln q_1 - 0.10 \ln q_2 - 0.08 \ln q_3 + 0.17 \ln q_4 + 0.08 \ln Q_4^*$$

Based on our analysis, the assumptions that the disturbances of the individual equations are independent and identically distributed, as well as contemporaneously correlated, are satisfied. As such, the SUR method of estimation provides a good fit for the individual equations as well as the overall system. This method provides valuable insights into the budgetary dynamics of the four commodities and serves as a useful tool for informing and guiding budgetary discussions.

Suggestions

Given the small sample size of 17 observations, the asymptotic properties of the parameter estimates may not be fully realized, as sample sizes greater than 30 are typically required to achieve efficient estimators. Therefore, a suggestion in obtaining a larger dataset would provide more justification for these methods. In addition, while the matrix of residual correlations showed slightly contemporaneously correlated error terms, they were not deemed overtly correlated. As such, an alternative approach could involve using Weighted Least Squares (WLS) estimation, due to the unequal variances among the equations. This method allows for a more parsimonious model without the complexities used in SUR.

```
fitWLS <- systemfit(system, method = "WLS", data = data)
summary(fitWLS, residCov = FALSE, equations = FALSE)
```

```
##
## systemfit results
## method: WLS
##
##         N DF      SSR detRCov   OLS-R2 McElroy-R2
## system 68 44 0.124978       0 0.869419   0.996972
##
##      N DF      SSR      MSE     RMSE       R2   Adj R2
## eq1 17 11 0.002091 0.000190 0.013787 0.894963 0.847218
## eq2 17 11 0.052952 0.004814 0.069382 0.556887 0.355472
## eq3 17 11 0.021996 0.002000 0.044717 0.829859 0.752522
## eq4 17 11 0.047939 0.004358 0.066016 0.930362 0.898709
##
##
## Coefficients:
##                     Estimate   Std. Error  t value  Pr(>|t|)
## eq1_(Intercept) -0.064406138  0.099104382 -0.64988 0.5291066
## eq1_log_q1       0.026234993  0.008612343  3.04621 0.0111226 *
## eq1_log_q2      -0.000320224  0.008432297 -0.03798 0.9703873
## eq1_log_q3      -0.015752701  0.010517636 -1.49774 0.1623356
## eq1_log_q4       0.011456514  0.015423663  0.74279 0.4731748
## eq1_log_Q_star  -0.008638002  0.006029513 -1.43262 0.1797674
## eq2_(Intercept)  0.896676263  0.498732013  1.79791 0.0996625 .
## eq2_log_q1      -0.032723772  0.043340678 -0.75504 0.4660863
## eq2_log_q2       0.134368224  0.042434618  3.16648 0.0089748 **
## eq2_log_q3      -0.089746788  0.052928856 -1.69561 0.1180435
## eq2_log_q4      -0.041547808  0.077617904 -0.53529 0.6031052
## eq2_log_Q_star  -0.027802560  0.030342866 -0.91628 0.3791700
## eq3_(Intercept)  0.512485869  0.321437057  1.59436 0.1391626
## eq3_log_q1       0.028249529  0.027933438  1.01132 0.3335938
## eq3_log_q2      -0.022127811  0.027349475 -0.80908 0.4356223
## eq3_log_q3       0.150641992  0.034113101  4.41596 0.0010354 **
## eq3_log_q4      -0.171173182  0.050025405 -3.42173 0.0057058 **
## eq3_log_Q_star   0.017554704  0.019556237  0.89765 0.3885958
## eq4_(Intercept) -0.470731906  0.474535648 -0.99198 0.3425249
## eq4_log_q1      -0.021658920  0.041237972 -0.52522 0.6098548
## eq4_log_q2      -0.099047278  0.040375870 -2.45313 0.0320672 *
## eq4_log_q3      -0.076459794  0.050360972 -1.51824 0.1571590
## eq4_log_q4       0.166095062  0.073852212  2.24902 0.0459689 *
## eq4_log_Q_star   0.082479563  0.028870759  2.85685 0.0156042 *
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The WLS method of parameter estimation yields the same individual R^2 values for each equation, as well as the same McElroy R^2, indicating a comparable goodness of fit to that of the SUR method.

```r
AICc(fitWLS)
```

```
## [1] -273.4669
```

```r
AICc(fitSUR)
```

```
## [1] -230.9867
```

Lastly, given the small sample size, the corrected Akaike Information Criterion (AICc) for WLS estimation is lower (-273.4669) compared to SUR (-230.9867), indicating that WLS provides a better model under these conditions. However, this comes at the cost of excluding the dynamics of correlated error terms captured by SUR. With a larger dataset, the presence of correlated error terms may become more apparent, further strengthening the justification for using SUR.